K-3 FORMATIVE ASSESSMENT CONSORTIUM

Mathematical Patterns

Construct Progression

DOMAIN: Cognitive Development

CLAIM: Students can use content-independent abilities and strategies as well as content-specific skills, processes, and approaches to solve problems and acquire information.

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Background Information
The purpose of this construct progression is to determine a child's level of understanding related to the structure of sequential and growing patterns. For sequential patterns, the focus is on duplicating, extending, and abstracting patterns, as well as identifying the repeating unit in patterns. For growing patterns, the focus is on applying, communicating, and symbolizing the rules that govern those patterns and relationships. The focus is not on a child's ability to accurately count objects or perform calculations. Mathematical patterns are also reflected in state Early Learning and Development Standards (ELDS) as well as K – 3 standards. For example, recognizing and extending patterns, creating patterns, and describing/analyzing patterns were identified as competencies reflected in the Consortium states' ELDS. Specifically, North Carolina's Foundations for Early Learning and Development includes the following standard: • Goal CD-11: Children compare, sort, group, organize, and measure objects and create patterns in their everyday environment. The importance of mathematical patterns in children's understanding of mathematical relationships is also reflected in the Common Core State Standards, both implicitly as building blocks in Grades K-2 and more explicitly in Grades 3 and 4. For example, the following CCSS are reflected in this progression: • K.OA.1— Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. • Grades 1 & 2 — Conduct various mathematical operations by "using objects, drawings, and equations with a symbol for the unknown number to represent the problem." • 3.OA.9—Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. • 4.OA.5—Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Additionally, in the CCSS, eight mathematical practices are identified that cut across standards for students at all grade levels. Five of these mathematical practices apply directly to mathematical patterns: • CCSS.Math.Practice.MP2 – Reason abstractly and quantitatively. • CCSS.Math.Practice.MP3 – Construct viable arguments and critique the reasoning of others. • CCSS.Math.Practice.MP4 – Model with mathematics. • CCSS.Math.Practice.MP5 – Use appropriate tools strategically. • CCSS.Math.Practice.MP7 – Look for and make use of structure. These practices are especially relevant in the later skills in the progression, which include children communicating about and symbolizing relationships between two variables.

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Rationale
Mathematical patterning is well established as a key competency in early mathematical development and beyond. In fact, the detection, identification, duplication, extension, and creation of patterns is a skill/competency or topic area covered in early curricula widely considered to be of high quality within the mathematics education research community (e.g., Big Ideas of Early Mathematics; The Young Child and Mathematics; Building Blocks). Additionally, patterning is integrated with established standards for the early years. More generally, research shows that math achievement measured around K entry is found to be highly predictive of subsequent mathematics achievement, measured around Grade 3 (Duncan et al., 2007; Claessens, Duncan, & Engel, 2009; Claessens & Engel, 2013). Key advocacy groups, such as the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM), have issued position statements on the importance of early mathematics, arguing that mathematics education for 3-to 6-year-olds is essential to promoting future mathematics achievement (NAEYC & NCTM, 2002). Children's ability in mathematics has also been found to affect reading ability. "Most surprising is that it also predicts later reading achievement even better than early reading skills. In fact, research shows that doing more mathematics increases oral language abilities, even when measured during the following school year. These include vocabulary, inference, independence, and grammatical complexity" (Clements & Sarama, 2013).

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Mathematical Patterns			
Understanding	**Skills**	**Performance Descriptors**	**Examples**
Children can look for and make use of structure in sequential patterns to understand they consist of repeating units that can be duplicated, extended, and abstracted.	A. Duplicates sequential AB patterns with the same materials.	Child duplicates at least 2 units of an AB pattern without any errors using identical materials.	*Mr. Garcia says to Luis, "Listen to this pattern" and makes the following model pattern: clap-stomp, clap-stomp. He then says to Luis, "Can you copy this pattern?" Luis makes the same pattern (clap-stomp, clap-stomp).*
	B. Duplicates sequential three-element patterns (e.g., ABB, ABC) with the same materials.	Child duplicates at least 2 units of a three-element pattern (e.g., ABB, ABC) without any errors using identical materials.	*During a study of bears, the class is reviewing the "Learned" column of their K-W-L* chart, including that most black bear mothers have two cubs each season. Mrs. Hoke uses large and small counting bears to make a pattern of large-small-small, large-small-small, large-small-small. Sonya duplicates the pattern with her own counting bears.* ** already know, want to know, and ultimately learn* *Mrs. Yang provides Diego with an ABB pattern made of orange and blue blocks and says, "Can you make this same pattern?" Diego selects an orange block from a pile of multicolored blocks and places it in front of him. Then he places two blue blocks next to it. Diego continues on until he has duplicated the entire pattern.*
	C. Extends sequential AB patterns at least one unit.	Child adds at least one complete unit to one end of an AB pattern OR verbally describes at least one complete unit without making any errors.	*Some friends have been playing hopscotch on the playground. When the teacher walks by, Cora says, "To get all the way to the end, my feet go one, two, one, two, one, two. If we want to make it longer, we can add another one and another two!"* *After studying the American flag, Gervase and Axl decide to make their own flag. Gervase paints a sequence of six alternating purple and yellow stripes. Axl then adds three more stripes (purple-yellow-purple).*

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	D. Extends sequential three-element patterns (e.g., ABB, ABC) at least one unit.	Child adds at least one complete unit to one end of a three-element pattern OR verbally describes at least one complete unit without making any errors.	*Phillip and Beth are playing a "snake-making game" with interlocking plastic cubes. Beth makes a snake with six cubes using green-green- red, green-green-red. She passes the snake to Phillip, who adds three more cubes (green-green-red) to the end.*
	E. Duplicates sequential patterns (e.g., AB, ABC, AABB) using materials different from those used in the model pattern (pattern abstraction).	Child duplicates at least one unit of a sequential pattern using DIFFERENT materials* without making any errors. ***Different materials:** Materials that are non-identical to those used in the model pattern. Different materials may be the same type of objects, as long as the attribute that is used to make the pattern changes. More commonly, different materials will be different types of objects altogether. (From Assessment Means Form)	*Mr. Garcia says to Beckett, "I am going to make a pattern with these blocks and ask you to copy the pattern by making sounds with your body like we have done with the other patterns." He says, "Look at this pattern" while making the following pattern with the blocks: blue-green, blue-green. Mr. Garcia then says to Beckett, "Now you copy this pattern by making sounds with your body." Beckett copies the pattern with the action pattern: clap-snap, clap-snap.* *During art class, Jacob says to the teacher, "I made the same kind of pattern as you. I have circle-square, circle-square on my paper." The teacher asks, "What do you mean the same kind of pattern as me?" Jacob replies, "Your shirt has a pattern of red-green, red-green stripes."*
	F. Identifies repeating unit in sequential patterns.	Child identifies (e.g., circles, points to, describes) a single unit of a sequential pattern.	*During a study of bears, the class is reviewing the "Learned" column of their K-W-L chart. After the teacher notes that most black bear mothers have two cubs each season, Ms. Bryan asks one child to come up front and stand, then two more children to kneel next to the first child. She then repeats the pattern with three more children. Maria says, "That's an ABB pattern with mama-baby-baby." During a music activity in which the children clap their hands and stomp their feet along with the music, Diego says, "The pattern is clap-stomp-stomp."*

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			Mrs. Young shows Stella a model ABC pattern using colored chips and says, "What is the group of chips that you see repeating one after the other?" Stella says, "I see yellow-green-blue repeating."
Children can look for and make use of structure in growing patterns to understand they are governed by rules that can be represented verbally and symbolically.	G. Extends by at least one step or determines the missing step in spatial or numerical growing patterns.	Child determines (e.g., says, draws) the missing or next step in a spatial or numerical growing pattern.	*EXTENDING - Remy is building a set of stairs with one-inch cubes [a spatial growing pattern]. He has reached four cubes high when Morrison sits down with him. Morrison asks if he can build too, and uses cubes to build the next step five cubes high. The spatial growing pattern here is a series of 1, 2, 3, 4 and 5 block-high stacks.* *DETERMINING A MISSING STEP - Mrs. Young makes a numerical growing pattern and deliberately leaves out the third step (2, 4, space, 8, 10). She asks Masha, "Does this look right to you? Please help me fix my growing pattern." Masha fills in the third step of the pattern with the number six.*
	H. Communicates a recursive rule governing the next step in spatial or numerical growing patterns.	Child communicates how a growing pattern changes from one step to the next [recursive rule].	*Alik and Denise are working with blocks. Alik is making a series of block towers where the 1st tower has three blocks, the 2nd tower has five blocks, and the 3rd tower has seven blocks. Denise says, "Oh, look - you're adding two every time."*
	I. Creates or enters data into a t-chart to document the relationship between the ordinal position of a step in a growing pattern (i.e., first, second, third) and an important feature of the step.	Child enters data into a t-chart (either created by the child or provided as a template) with the ordinal position of each step in a growing pattern in the 1st column and the values of the changing feature in the 2nd column.	*CHILD GIVEN A BLANK T-CHART - Mrs. Young shows Alice a numerical growing pattern: 10, 20, 30. Mrs. Young gives Alice an empty t-chart and says, "Can you fill in this t-chart to show how this pattern is changing?" Alice enters the following information in the t-chart:*

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<table>
<tr><th colspan="4">Mathematical Patterns</th></tr>
<tr><th>Understanding</th><th>Skills</th><th>Performance Descriptors</th><th>Examples</th></tr>
<tr><td rowspan="3"></td><td></td><td></td><td><table><tr><th>Step</th><th>Number</th></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>20</td></tr><tr><td>3</td><td>30</td></tr></table></td></tr>
<tr><td>J. <u>Applies the relationship</u> between the two variables in a <u>t-chart</u> to extend a numerical growing pattern by at least one step.</td><td>Child extends a numerical growing pattern in a t-chart by at least one step using a functional rule [NOTE: Teachers can provide the ordinal position for the next step].</td><td>During a study of bears, the class has made a t-chart to show that if one mama bear has two cubs, two mama bears will have four cubs, and three mama bears will have six cubs. Then the teacher writes a four in the 1st column:<table><tr><th>Mama bears</th><th>Cubs</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr><tr><td>4</td><td></td></tr></table>The teacher says, "How many cubs will there be when there are four mama bears?" Charisse says, "You multiply the number of Mama bears times two. So, there will be eight cubs. That's a lot of cubs!"</td></tr>
<tr><td>K. Communicates a <u>one-operation functional rule</u> governing spatial or numerical growing patterns and uses it to determine a <u>far step</u>. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.</td><td>Child describes a one-operation functional rule for a growing pattern AND uses the rule to determine a far step in the pattern NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.</td><td>The class is planning for field day. The teacher says, "There will be 16 stations and each station can have 10 children." The teacher writes a row of numbers (10, 20, 30) on the board and says, "One station can have 10 children, two stations can have 20 children, and three stations can have 30 children. How could we find out how many children can participate in all 16 stations."? Jake says, "We can multiply the number of stations by ten to find out how many children can play. 16 times 10 is 160, so 160 children can participate."</td></tr>
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Understanding	**Skills**	**Performance Descriptors**	**Examples**
	L. Communicates a <u>two-operation functional rule</u> governing spatial or numerical growing patterns and uses it to determine a <u>far step</u>. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.	Child describes a two-operation functional rule for a growing pattern AND uses the rule to determine a far step in the pattern NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.	*Mrs. Young presents Ricky with a spatial growing pattern (3 pentagons, 5 pentagons, 7 pentagons, 9 pentagons) and says, "Tell me the rule for finding the value of any step in the pattern." Ricky says, "For each step, the pentagons increase by two. First, I should multiply 2 times the step. Then, I have to add the pentagon on the top. So, the rule is 2 times the step plus 1." Mrs. Young then says, "How many pentagons would be in the 14th step in the pattern?" Ricky says, "The 14th step would be 14 times 2 plus 1. That is 29."*
	M. <u>Creates an equation</u> that symbolizes a <u>functional rule</u> governing a spatial or numerical growing pattern.	Child generates (e.g., writes, dictates) an equation that symbolizes a functional rule for a spatial or numerical growing pattern.	*Mrs. Young presents Rejane with a spatial growing pattern (3 pentagons, 5 pentagons, 7 pentagons, 9 pentagons) along with a completed t-chart and says, "Write an equation that shows the rule for finding the value of any step in the pattern." Rejane writes the following equation: Pentagons = 2 X Step + 1.*

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RESOURCES

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